PRECALCULUS

MATHEMATICS FOR CALCULUS

SEVENTH EDITION

James Stewart | Lothar Redlin | Saleem Watson

EXPONENTS AND RADICALS

$x^m x^n = x^{m+n}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$x^{-n} = \dfrac{1}{x^n}$

$(xy)^n = x^n y^n$

$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$

$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$

$\sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$

$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$

SPECIAL PRODUCTS

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

FACTORING FORMULAS

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$.

$|x| < a$ means $-a < x < a$.

$|x| > a$ means $x > a$ or $x < -a$.

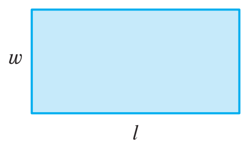
GEOMETRIC FORMULAS

Formulas for area A, perimeter P, circumference C, volume V:

Rectangle

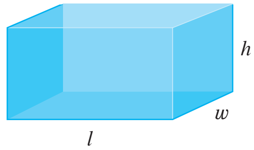
$A = lw$

$P = 2l + 2w$



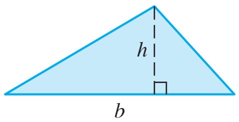
Box

$V = lwh$



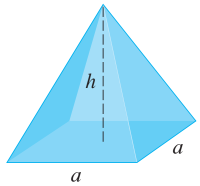
Triangle

$A = \frac{1}{2}bh$



Pyramid

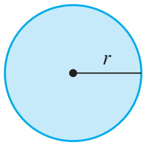
$V = \frac{1}{3}ha^2$



Circle

$A = \pi r^2$

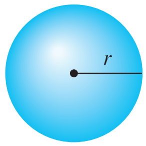
$C = 2\pi r$



Sphere

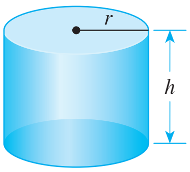
$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$



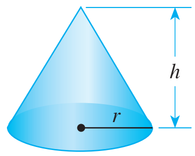
Cylinder

$V = \pi r^2 h$



Cone

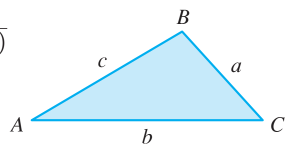
$V = \frac{1}{3}\pi r^2 h$



HERON'S FORMULA

Area $= \sqrt{s(s - a)(s - b)(s - c)}$

where $s = \dfrac{a + b + c}{2}$



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DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

LINES

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m $\qquad y - y_1 = m(x - x_1)$

Slope-intercept equation of line with slope m and y-intercept b $\qquad y = mx + b$

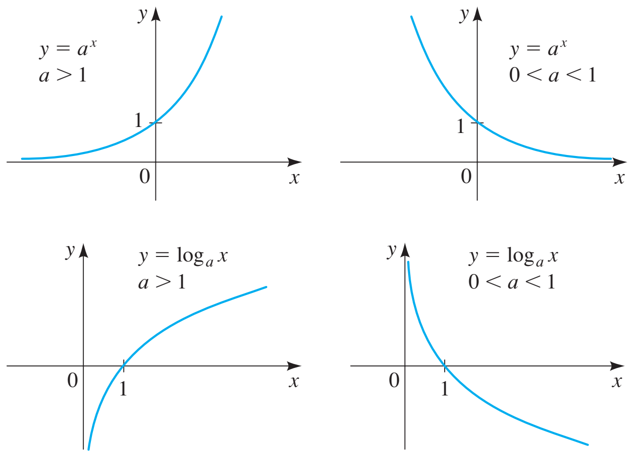
Two-intercept equation of line with x-intercept a and y-intercept b $\qquad \dfrac{x}{a} + \dfrac{y}{b} = 1$

LOGARITHMS

$y = \log_a x \quad$ means $\quad a^y = x$

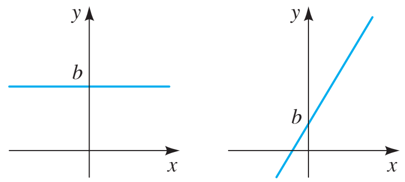
$\log_a a^x = x$	$a^{\log_a x} = x$
$\log_a 1 = 0$	$\log_a a = 1$
$\log x = \log_{10} x$	$\ln x = \log_e x$
$\log_a xy = \log_a x + \log_a y$	$\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$
$\log_a x^b = b \log_a x$	$\log_b x = \dfrac{\log_a x}{\log_a b}$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS



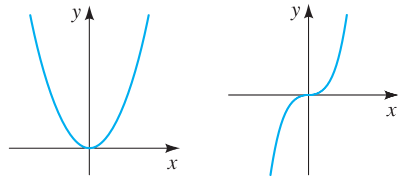
GRAPHS OF FUNCTIONS

Linear functions: $f(x) = mx + b$



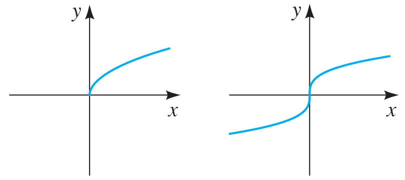
$f(x) = b \qquad f(x) = mx + b$

Power functions: $f(x) = x^n$



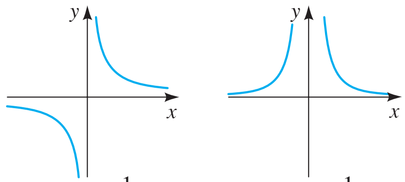
$f(x) = x^2 \qquad f(x) = x^3$

Root functions: $f(x) = \sqrt[n]{x}$



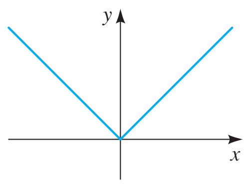
$f(x) = \sqrt{x} \qquad f(x) = \sqrt[3]{x}$

Reciprocal functions: $f(x) = 1/x^n$



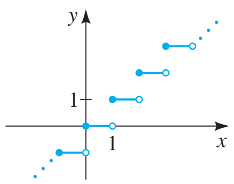
$f(x) = \dfrac{1}{x} \qquad f(x) = \dfrac{1}{x^2}$

Absolute value function



$f(x) = |x|$

Greatest integer function



$f(x) = [\![x]\!]$

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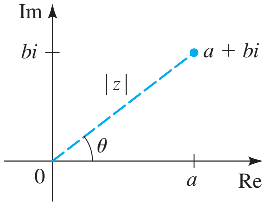
COMPLEX NUMBERS

For the complex number $z = a + bi$

the **conjugate** is $\bar{z} = a - bi$

the **modulus** is $|z| = \sqrt{a^2 + b^2}$

the **argument** is θ, where $\tan\theta = b/a$



Polar form of a complex number

For $z = a + bi$, the **polar form** is

$$z = r(\cos\theta + i\sin\theta)$$

where $r = |z|$ is the modulus of z and θ is the argument of z

De Moivre's Theorem

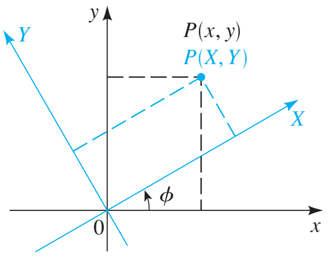
$$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$\sqrt[n]{z} = [r(\cos\theta + i\sin\theta)]^{1/n}$$

$$= r^{1/n}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right)$$

where $k = 0, 1, 2, \ldots, n - 1$

ROTATION OF AXES



Rotation of axes formulas

$$x = X\cos\phi - Y\sin\phi$$
$$y = X\sin\phi + Y\cos\phi$$

Angle-of-rotation formula for conic sections

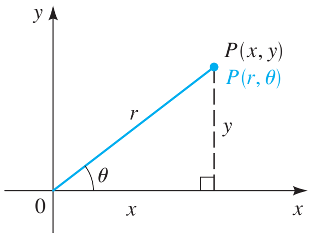
To eliminate the xy-term in the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

rotate the axis by the angle ϕ that satisfies

$$\cot 2\phi = \frac{A - C}{B}$$

POLAR COORDINATES



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$

SUMS OF POWERS OF INTEGERS

$$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n + 1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n + 1)^2}{4}$$

THE DERIVATIVE

The **average rate of change** of f between a and b is

$$\frac{f(b) - f(a)}{b - a}$$

The **derivative** of f at a is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

AREA UNDER THE GRAPH OF *f*

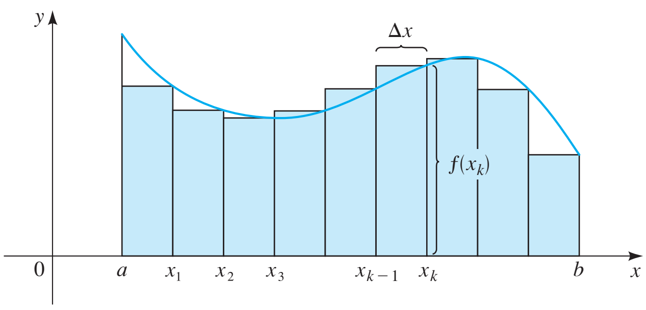
The **area under the graph of *f*** on the interval $[a, b]$ is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

where

$$\Delta x = \frac{b - a}{n}$$

$$x_k = a + k\,\Delta x$$



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ABOUT THE AUTHORS

JAMES STEWART received his MS from Stanford University and his PhD from the University of Toronto. He did research at the University of London and was influenced by the famous mathematician George Polya at Stanford University. Stewart is Professor Emeritus at McMaster University and is currently Professor of Mathematics at the University of Toronto. His research field is harmonic analysis and the connections between mathematics and music. James Stewart is the author of a bestselling calculus textbook series published by Cengage Learning, including *Calculus, Calculus: Early Transcendentals,* and *Calculus: Concepts and Contexts;* a series of precalculus texts; and a series of high-school mathematics textbooks.

LOTHAR REDLIN grew up on Vancouver Island, received a Bachelor of Science degree from the University of Victoria, and received a PhD from McMaster University in 1978. He subsequently did research and taught at the University of Washington, the University of Waterloo, and California State University, Long Beach. He is currently Professor of Mathematics at The Pennsylvania State University, Abington Campus. His research field is topology.

SALEEM WATSON received his Bachelor of Science degree from Andrews University in Michigan. He did graduate studies at Dalhousie University and McMaster University, where he received his PhD in 1978. He subsequently did research at the Mathematics Institute of the University of Warsaw in Poland. He also taught at The Pennsylvania State University. He is currently Professor of Mathematics at California State University, Long Beach. His research field is functional analysis.

Stewart, Redlin, and Watson have also published *College Algebra, Trigonometry, Algebra and Trigonometry,* and (with Phyllis Panman) *College Algebra: Concepts and Contexts.*

ABOUT THE COVER

The cover photograph shows a bridge in Valencia, Spain, designed by the Spanish architect Santiago Calatrava. The bridge leads to the Agora Stadium, also designed by Calatrava, which was completed in 2009 to host the Valencia Open tennis tournament. Calatrava has always been very interested in how mathematics can help him realize the buildings he imagines. As a young student, he taught himself descriptive geometry from books in order to represent three-dimensional objects in two dimensions. Trained as both an engineer and an architect, he wrote a doctoral thesis in 1981 entitled "On the Foldability of Space Frames," which is filled with mathematics, especially geometric transformations. His strength as an engineer enables him to be daring in his architecture.

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MATHEMATICS FOR CALCULUS

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Photo Researcher: Lumina Datamatics Ltd.

Text Researcher: Lumina Datamatics Ltd.

Copy Editor: Barbara Willette

Illustrator: Precision Graphics; Graphic World, Inc.

Text Designer: Diane Beasley

Cover Designer: Cheryl Carrington

Cover Image: AWL Images/Masterfile

Compositor: Graphic World, Inc.

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WCN: 02-200-203

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Library of Congress Control Number: 2014948805

Student Edition:
ISBN: 978-1-305-07175-9

Loose-leaf Edition:
ISBN: 978-1-305-58602-4

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Printed in the United States of America
Print Number: 01 Print Year: 2014

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PREFACE

What do students really need to know to be prepared for calculus? What tools do instructors really need to assist their students in preparing for calculus? These two questions have motivated the writing of this book.

To be prepared for calculus a student needs not only technical skill but also a clear understanding of concepts. Indeed, *conceptual understanding* and *technical skill* go hand in hand, each reinforcing the other. A student also needs to gain an appreciation for the power and utility of mathematics in *modeling* the real world. Every feature of this textbook is devoted to fostering these goals.

In this Seventh Edition our objective is to further enhance the effectiveness of the book as an instructional tool for teachers and as a learning tool for students. Many of the changes in this edition are a result of suggestions we received from instructors and students who are using the current edition; others are a result of insights we have gained from our own teaching. Some chapters have been reorganized and rewritten, new sections have been added (as described below), the review material at the end of each chapter has been substantially expanded, and exercise sets have been enhanced to further focus on the main concepts of precalculus. In all these changes and numerous others (small and large) we have retained the main features that have contributed to the success of this book.

New to the Seventh Edition

- **Exercises** More than 20% of the exercises are new, and groups of exercises now have headings that identify the type of exercise. New *Skills Plus* exercises in most sections contain more challenging exercises that require students to extend and synthesize concepts.
- **Review Material** The review material at the end of each chapter now includes a summary of *Properties and Formulas* and a new *Concept Check*. Each *Concept Check* provides a step-by-step review of all the main concepts and applications of the chapter. Answers to the *Concept Check* questions are on tear-out sheets at the back of the book.
- **Discovery Projects** References to *Discovery Projects,* including brief descriptions of the content of each project, are located in boxes where appropriate in each chapter. These boxes highlight the applications of precalculus in many different real-world contexts. (The projects are located at the book companion website: **www.stewartmath.com**.)
- **Geometry Review** A new Appendix A contains a review of the main concepts of geometry used in this book, including similarity and the Pythagorean Theorem.
- **CHAPTER 1 Fundamentals** This chapter now contains two new sections. Section 1.6, "Complex Numbers" (formerly in Chapter 3), has been moved here. Section 1.12, "Modeling Variation," is now also in this chapter.
- **CHAPTER 2 Functions** This chapter now includes the new Section 2.5, "Linear Functions and Models." This section highlights the connection between the slope of a line and the rate of change of a linear function. These two interpretations of slope help prepare students for the concept of the derivative in calculus.
- **CHAPTER 3 Polynomial and Rational Functions** This chapter now includes the new Section 3.7, "Polynomial and Rational Inequalities." Section 3.6, "Rational Functions," has a new subsection on rational functions with "holes." The sections on complex numbers and on variation have been moved to Chapter 1.

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- **CHAPTER 4 Exponential and Logarithmic Functions** The chapter now includes two sections on the applications of these functions. Section 4.6, "Modeling with Exponential Functions," focuses on modeling growth and decay, Newton's Law of Cooling, and other such applications. Section 4.7, "Logarithmic Scales," covers the concept of a logarithmic scale with applications involving the pH, Richter, and decibel scales.
- **CHAPTER 5 Trigonometric Functions: Unit Circle Approach** This chapter includes a new subsection on the concept of phase shift as used in modeling harmonic motion.
- **CHAPTER 10 Systems of Equations and Inequalities** The material on systems of inequalities has been rewritten to emphasize the steps used in graphing the solution of a system of inequalities.

Teaching with the Help of This Book

We are keenly aware that good teaching comes in many forms and that there are many different approaches to teaching and learning the concepts and skills of precalculus. The organization and exposition of the topics in this book are designed to accommodate different teaching and learning styles. In particular, each topic is presented algebraically, graphically, numerically, and verbally, with emphasis on the relationships between these different representations. The following are some special features that can be used to complement different teaching and learning styles:

Exercise Sets The most important way to foster conceptual understanding and hone technical skill is through the problems that the instructor assigns. To that end we have provided a wide selection of exercises.

- **Concept Exercises** These exercises ask students to use mathematical language to state fundamental facts about the topics of each section.
- **Skills Exercises** These exercises reinforce and provide practice with all the learning objectives of each section. They comprise the core of each exercise set.
- **Skills Plus Exercises** The *Skills Plus* exercises contain challenging problems that often require the synthesis of previously learned material with new concepts.
- **Applications Exercises** We have included substantial applied problems from many different real-world contexts. We believe that these exercises will capture students' interest.
- **Discovery, Writing, and Group Learning** Each exercise set ends with a block of exercises labeled *Discuss* ■ *Discover* ■ *Prove* ■ *Write.* These exercises are designed to encourage students to experiment, preferably in groups, with the concepts developed in the section and then to write about what they have learned rather than simply looking for the answer. New *Prove* exercises highlight the importance of deriving a formula.
- **Now Try Exercise . . .** At the end of each example in the text the student is directed to one or more similar exercises in the section that help to reinforce the concepts and skills developed in that example.
- **Check Your Answer** Students are encouraged to check whether an answer they obtained is reasonable. This is emphasized throughout the text in numerous *Check Your Answer* sidebars that accompany the examples (see, for instance, pages 54 and 71).

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A Complete Review Chapter We have included an extensive review chapter primarily as a handy reference for the basic concepts that are preliminary to this course.

- **CHAPTER 1 Fundamentals** This is the review chapter; it contains the fundamental concepts from algebra and analytic geometry that a student needs in order to begin a precalculus course. As much or as little of this chapter can be covered in class as needed, depending on the background of the students.
- **CHAPTER 1 Test** The test at the end of Chapter 1 is designed as a diagnostic test for determining what parts of this review chapter need to be taught. It also serves to help students gauge exactly what topics they need to review.

Flexible Approach to Trigonometry The trigonometry chapters of this text have been written so that either the right triangle approach or the unit circle approach may be taught first. Putting these two approaches in different chapters, each with its relevant applications, helps to clarify the purpose of each approach. The chapters introducing trigonometry are as follows.

- **CHAPTER 5 Trigonometric Functions: Unit Circle Approach** This chapter introduces trigonometry through the unit circle approach. This approach emphasizes that the trigonometric functions are functions of real numbers, just like the polynomial and exponential functions with which students are already familiar.
- **CHAPTER 6 Trigonometric Functions: Right Triangle Approach** This chapter introduces trigonometry through the right triangle approach. This approach builds on the foundation of a conventional high-school course in trigonometry.

Another way to teach trigonometry is to intertwine the two approaches. Some instructors teach this material in the following order: Sections 5.1, 5.2, 6.1, 6.2, 6.3, 5.3, 5.4, 5.5, 5.6, 6.4, 6.5, and 6.6. Our organization makes it easy to do this without obscuring the fact that the two approaches involve distinct representations of the same functions.

Graphing Calculators and Computers We make use of graphing calculators and computers in examples and exercises throughout the book. Our calculator-oriented examples are always preceded by examples in which students must graph or calculate by hand so that they can understand precisely what the calculator is doing when they later use it to simplify the routine, mechanical part of their work. The graphing calculator sections, subsections, examples, and exercises, all marked with the special symbol , are optional and may be omitted without loss of continuity.

- **Using a Graphing Calculator** General guidelines on using graphing calculators and a quick reference guide to using TI-83/84 calculators are available at the book companion website: **www.stewartmath.com**.
- **Graphing, Regression, Matrix Algebra** Graphing calculators are used throughout the text to graph and analyze functions, families of functions, and sequences; to calculate and graph regression curves; to perform matrix algebra; to graph linear inequalities; and other powerful uses.
- **Simple Programs** We exploit the programming capabilities of a graphing calculator to simulate real-life situations, to sum series, or to compute the terms of a recursive sequence (see, for instance, pages 628, 896, and 939).

Focus on Modeling The theme of modeling has been used throughout to unify and clarify the many applications of precalculus. We have made a special effort to clarify the essential process of translating problems from English into the language of mathematics (see pages 238 and 686).

- **Constructing Models** There are many applied problems throughout the book in which students are given a model to analyze (see, for instance, page 250). But the material on modeling, in which students are required to *construct* mathematical models, has been organized into clearly defined sections and subsections (see, for instance, pages 370, 445, and 685).

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- **Focus on Modeling** Each chapter concludes with a *Focus on Modeling* section. The first such section, after Chapter 1, introduces the basic idea of modeling a real-life situation by fitting lines to data (linear regression). Other sections present ways in which polynomial, exponential, logarithmic, and trigonometric functions, and systems of inequalities can all be used to model familiar phenomena from the sciences and from everyday life (see, for instance, pages 325, 392, and 466).

Review Sections and Chapter Tests Each chapter ends with an extensive review section that includes the following.

- **Properties and Formulas** The *Properties and Formulas* at the end of each chapter contains a summary of the main formulas and procedures of the chapter (see, for instance, pages 386 and 460).
- **Concept Check and Concept Check Answers** The *Concept Check* at the end of each chapter is designed to get the students to think about and explain each concept presented in the chapter and then to use the concept in a given problem. This provides a step-by-step review of all the main concepts in a chapter (see, for instance, pages 230, 319, and 769). Answers to the *Concept Check* questions are on tear-out sheets at the back of the book.
- **Review Exercises** The *Review Exercises* at the end of each chapter recapitulate the basic concepts and skills of the chapter and include exercises that combine the different ideas learned in the chapter.
- **Chapter Test** Each review section concludes with a Chapter Test designed to help students gauge their progress.
- **Cumulative Review Tests** *Cumulative Review Tests* following selected chapters are available at the book companion website. These tests contain problems that combine skills and concepts from the preceding chapters. The problems are designed to highlight the connections between the topics in these related chapters.
- **Answers** Brief answers to odd-numbered exercises in each section (including the review exercises) and to all questions in the *Concepts* exercises and *Chapter Tests,* are given in the back of the book.

Mathematical Vignettes Throughout the book we make use of the margins to provide historical notes, key insights, or applications of mathematics in the modern world. These serve to enliven the material and show that mathematics is an important, vital activity and that even at this elementary level it is fundamental to everyday life.

- **Mathematical Vignettes** These vignettes include biographies of interesting mathematicians and often include a key insight that the mathematician discovered (see, for instance, the vignettes on Viète, page 50; Salt Lake City, page 93; and radiocarbon dating, page 367).
- **Mathematics in the Modern World** This is a series of vignettes that emphasize the central role of mathematics in current advances in technology and the sciences (see, for instance, pages 302, 753, and 784).

Book Companion Website A website that accompanies this book is located at **www.stewartmath.com**. The site includes many useful resources for teaching precalculus, including the following.

- **Discovery Projects** *Discovery Projects* for each chapter are available at the book companion website. The projects are referenced in the text in the appropriate sections. Each project provides a challenging yet accessible set of activities that enable students (perhaps working in groups) to explore in greater depth an interesting aspect of the topic they have just learned (see, for instance, the Discovery Projects *Visualizing a Formula, Relations and Functions, Will the Species Survive?,* and *Computer Graphics I* and *II*, referenced on pages 29, 163, 719, 738, and 820).

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- **Focus on Problem Solving** Several *Focus on Problem Solving* sections are available on the website. Each such section highlights one of the problem-solving principles introduced in the Prologue and includes several challenging problems (see for instance *Recognizing Patterns, Using Analogy, Introducing Something Extra, Taking Cases,* and *Working Backward*).
- **Cumulative Review Tests** *Cumulative Review Tests* following Chapters 4, 7, 9, 11, and 13 are available on the website.
- **Appendix B: Calculations and Significant Figures** This appendix, available at the book companion website, contains guidelines for rounding when working with approximate values.
- **Appendix C: Graphing with a Graphing Calculator** This appendix, available at the book companion website, includes general guidelines on graphing with a graphing calculator as well as guidelines on how to avoid common graphing pitfalls.
- **Appendix D: Using the TI-83/84 Graphing Calculator** In this appendix, available at the book companion website, we provide simple, easy-to-follow, step-by-step instructions for using the TI-83/84 graphing calculators.

Acknowledgments

We feel fortunate that all those involved in the production of this book have worked with exceptional energy, intense dedication, and passionate interest. It is surprising how many people are essential in the production of a mathematics textbook, including content editors, reviewers, faculty colleagues, production editors, copy editors, permissions editors, solutions and accuracy checkers, artists, photo researchers, text designers, typesetters, compositors, proofreaders, printers, and many more. We thank them all. We particularly mention the following.

Reviewers for the Sixth Edition Raji Baradwaj, UMBC; Chris Herman, Lorain County Community College; Irina Kloumova, Sacramento City College; Jim McCleery, Skagit Valley College, Whidbey Island Campus; Sally S. Shao, Cleveland State University; David Slutzky, Gainesville State College; Edward Stumpf, Central Carolina Community College; Ricardo Teixeira, University of Texas at Austin; Taixi Xu, Southern Polytechnic State University; and Anna Wlodarczyk, Florida International University.

Reviewers for the Seventh Edition Mary Ann Teel, University of North Texas; Natalia Kravtsova, The Ohio State University; Belle Sigal, Wake Technical Community College; Charity S. Turner, The Ohio State University; Yu-ing Hargett, Jefferson State Community College–Alabama; Alicia Serfaty de Markus, Miami Dade College; Cathleen Zucco-Teveloff, Rider University; Minal Vora, East Georgia State College; Sutandra Sarkar, Georgia State University; Jennifer Denson, Hillsborough Community College; Candice L. Ridlon, University of Maryland Eastern Shore; Alin Stancu, Columbus State University; Frances Tishkevich, Massachusetts Maritime Academy; Phil Veer, Johnson County Community College; Cathleen Zucco-Teveloff, Rider University; Phillip Miller, Indiana University–Southeast; Mildred Vernia, Indiana University–Southeast; Thurai Kugan, John Jay College–CUNY.

We are grateful to our colleagues who continually share with us their insights into teaching mathematics. We especially thank Robert Mena at California State University, Long Beach; we benefited from his many insights into mathematics and its history. We thank Cecilia McVoy at Penn State Abington for her helpful suggestions. We thank Andrew Bulman-Fleming for writing the Solutions Manual and Doug Shaw at the University of Northern Iowa for writing the Instructor Guide and the Study Guide. We are very grateful to Frances Gulick at the University of Maryland for checking the accuracy of the entire manuscript and doing each and every exercise; her many suggestions and corrections have contributed greatly to the accuracy and consistency of the contents of this book.

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We thank Martha Emry, our production service and art editor; her energy, devotion, and experience are essential components in the creation of this book. We are grateful for her remarkable ability to instantly recall, when needed, any detail of the entire manuscript as well as her extraordinary ability to simultaneously manage several interdependent editing tracks. We thank Barbara Willette, our copy editor, for her attention to every detail in the manuscript and for ensuring a consistent, appropriate style throughout the book. We thank our designer, Diane Beasley, for the elegant and appropriate design for the interior of the book. We thank Graphic World for their attractive and accurate graphs and Precision Graphics for bringing many of our illustrations to life. We thank our compositors at Graphic World for ensuring a balanced and coherent look for each page of the book.

At Cengage Learning we thank Jennifer Risden, content project manager, for her professional management of the production of the book. We thank Lynh Pham, media developer, for his expert handling of many technical issues, including the creation of the book companion website. We thank Vernon Boes, art director, for his capable administration of the design of the book. We thank Mark Linton, marketing manager, for helping bring the book to the attention of those who may wish to use it in their classes.

We particularly thank our developmental editor, Stacy Green, for skillfully guiding and facilitating every aspect of the creation of this book. Her interest in the book, her familiarity with the entire manuscript, and her almost instant responses to our many queries have made the writing of the book an even more enjoyable experience for us.

Above all we thank our acquisitions editor, Gary Whalen. His vast editorial experience, his extensive knowledge of current issues in the teaching of mathematics, his skill in managing the resources needed to enhance this book, and his deep interest in mathematics textbooks have been invaluable assets in the creation of this book.

Ancillaries

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Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via **www.cengage.com/login**. Access and download PowerPoint presentations, images, instructor's manual, and more.

Complete Solutions Manual
The Complete Solutions Manual provides worked-out solutions to all of the problems in the text. Located on the companion website.

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The Test Bank provides chapter tests and final exams, along with answer keys. Located on the companion website.

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The Instructor's Guide contains points to stress, suggested time to allot, text discussion topics, core materials for lecture, workshop/discussion suggestions, group work exercises in a form suitable for handout, and suggested homework problems. Located on the companion website.

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TO THE STUDENT

This textbook was written for you to use as a guide to mastering precalculus mathematics. Here are some suggestions to help you get the most out of your course.

First of all, you should read the appropriate section of text *before* you attempt your homework problems. Reading a mathematics text is quite different from reading a novel, a newspaper, or even another textbook. You may find that you have to reread a passage several times before you understand it. Pay special attention to the examples, and work them out yourself with pencil and paper as you read. Then do the linked exercises referred to in *"Now Try Exercise . . ."* at the end of each example. With this kind of preparation you will be able to do your homework much more quickly and with more understanding.

Don't make the mistake of trying to memorize every single rule or fact you may come across. Mathematics doesn't consist simply of memorization. Mathematics is a *problem-solving art*, not just a collection of facts. To master the subject you must solve problems—lots of problems. Do as many of the exercises as you can. Be sure to write your solutions in a logical, step-by-step fashion. Don't give up on a problem if you can't solve it right away. Try to understand the problem more clearly—reread it thoughtfully and relate it to what you have learned from your teacher and from the examples in the text. Struggle with it until you solve it. Once you have done this a few times you will begin to understand what mathematics is really all about.

Answers to the odd-numbered exercises, as well as all the answers (even and odd) to the concept exercises and chapter tests, appear at the back of the book. If your answer differs from the one given, don't immediately assume that you are wrong. There may be a calculation that connects the two answers and makes both correct. For example, if you get $1/(\sqrt{2} - 1)$ but the answer given is $1 + \sqrt{2}$, your answer *is* correct, because you can multiply both numerator and denominator of your answer by $\sqrt{2} + 1$ to change it to the given answer. In rounding approximate answers, follow the guidelines in Appendix B: *Calculations and Significant Figures.*

The symbol ⊘ is used to warn against committing an error. We have placed this symbol in the margin to point out situations where we have found that many of our students make the same mistake.

Abbreviations

The following abbreviations are used throughout the text.

cm	centimeter	**kPa**	kilopascal	**N**	Newton
dB	decibel	**L**	liter	**qt**	quart
F	farad	**lb**	pound	**oz**	ounce
ft	foot	**lm**	lumen	**s**	second
g	gram	**M**	mole of solute	**Ω**	ohm
gal	gallon		per liter of	**V**	volt
h	hour		solution	**W**	watt
H	henry	**m**	meter	**yd**	yard
Hz	Hertz	**mg**	milligram	**yr**	year
in.	inch	**MHz**	megahertz	**°C**	degree Celsius
J	Joule	**mi**	mile	**°F**	degree Fahrenheit
kcal	kilocalorie	**min**	minute	**K**	Kelvin
kg	kilogram	**mL**	milliliter	⇒	implies
km	kilometer	**mm**	millimeter	⇔	is equivalent to

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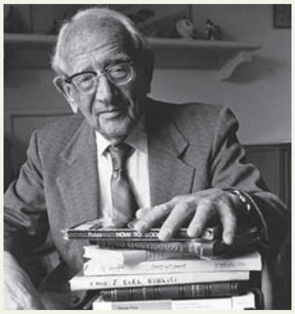
PROLOGUE PRINCIPLES OF PROBLEM SOLVING



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GEORGE POLYA (1887–1985) is famous among mathematicians for his ideas on problem solving. His lectures on problem solving at Stanford University attracted overflow crowds whom he held on the edges of their seats, leading them to discover solutions for themselves. He was able to do this because of his deep insight into the psychology of problem solving. His well-known book *How To Solve It* has been translated into 15 languages. He said that Euler (see page 63) was unique among great mathematicians because he explained how he found his results. Polya often said to his students and colleagues, "Yes, I see that your proof is correct, but how did you discover it?" In the preface to *How To Solve It,* Polya writes, "A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery."

The ability to solve problems is a highly prized skill in many aspects of our lives; it is certainly an important part of any mathematics course. There are no hard and fast rules that will ensure success in solving problems. However, in this Prologue we outline some general steps in the problem-solving process and we give principles that are useful in solving certain types of problems. These steps and principles are just common sense made explicit. They have been adapted from George Polya's insightful book *How To Solve It*.

1. Understand the Problem

The first step is to read the problem and make sure that you understand it. Ask yourself the following questions:

What is the unknown?

What are the given quantities?

What are the given conditions?

For many problems it is useful to

draw a diagram

and identify the given and required quantities on the diagram. Usually, it is necessary to

introduce suitable notation

In choosing symbols for the unknown quantities, we often use letters such as a, b, c, m, n, x, and y, but in some cases it helps to use initials as suggestive symbols, for instance, V for volume or t for time.

2. Think of a Plan

Find a connection between the given information and the unknown that enables you to calculate the unknown. It often helps to ask yourself explicitly: "How can I relate the given to the unknown?" If you don't see a connection immediately, the following ideas may be helpful in devising a plan.

■ Try to Recognize Something Familiar

Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown.

■ Try to Recognize Patterns

Certain problems are solved by recognizing that some kind of pattern is occurring. The pattern could be geometric, numerical, or algebraic. If you can see regularity or repetition in a problem, then you might be able to guess what the pattern is and then prove it.

■ Use Analogy

Try to think of an analogous problem, that is, a similar or related problem but one that is easier than the original. If you can solve the similar, simpler problem, then it might give you the clues you need to solve the original, more difficult one. For instance, if a problem involves very large numbers, you could first try a similar problem with smaller numbers. Or if the problem is in three-dimensional geometry, you could look for something similar in two-dimensional geometry. Or if the problem you start with is a general one, you could first try a special case.

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■ Introduce Something Extra

You might sometimes need to introduce something new—an auxiliary aid—to make the connection between the given and the unknown. For instance, in a problem for which a diagram is useful, the auxiliary aid could be a new line drawn in the diagram. In a more algebraic problem the aid could be a new unknown that relates to the original unknown.

■ Take Cases

You might sometimes have to split a problem into several cases and give a different argument for each case. For instance, we often have to use this strategy in dealing with absolute value.

■ Work Backward

Sometimes it is useful to imagine that your problem is solved and work backward, step by step, until you arrive at the given data. Then you might be able to reverse your steps and thereby construct a solution to the original problem. This procedure is commonly used in solving equations. For instance, in solving the equation $3x - 5 = 7$, we suppose that x is a number that satisfies $3x - 5 = 7$ and work backward. We add 5 to each side of the equation and then divide each side by 3 to get $x = 4$. Since each of these steps can be reversed, we have solved the problem.

■ Establish Subgoals

In a complex problem it is often useful to set subgoals (in which the desired situation is only partially fulfilled). If you can attain or accomplish these subgoals, then you might be able to build on them to reach your final goal.

■ Indirect Reasoning

Sometimes it is appropriate to attack a problem indirectly. In using **proof by contradiction** to prove that P implies Q, we assume that P is true and Q is false and try to see why this cannot happen. Somehow we have to use this information and arrive at a contradiction to what we absolutely know is true.

■ Mathematical Induction

In proving statements that involve a positive integer n, it is frequently helpful to use the Principle of Mathematical Induction, which is discussed in Section 12.5.

3. Carry Out the Plan

In Step 2, a plan was devised. In carrying out that plan, you must check each stage of the plan and write the details that prove that each stage is correct.

4. Look Back

Having completed your solution, it is wise to look back over it, partly to see whether any errors have been made and partly to see whether you can discover an easier way to solve the problem. Looking back also familiarizes you with the method of solution, which may be useful for solving a future problem. Descartes said, "Every problem that I solved became a rule which served afterwards to solve other problems."

We illustrate some of these principles of problem solving with an example.

PROBLEM ■ Average Speed

A driver sets out on a journey. For the first half of the distance, she drives at the leisurely pace of 30 mi/h; during the second half she drives 60 mi/h. What is her average speed on this trip?

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THINKING ABOUT THE PROBLEM

It is tempting to take the average of the speeds and say that the average speed for the entire trip is

$$\frac{30 + 60}{2} = 45 \text{ mi/h}$$

But is this simple-minded approach really correct?

Try a special case. ▶

Let's look at an easily calculated special case. Suppose that the total distance traveled is 120 mi. Since the first 60 mi is traveled at 30 mi/h, it takes 2 h. The second 60 mi is traveled at 60 mi/h, so it takes one hour. Thus, the total time is $2 + 1 = 3$ hours and the average speed is

$$\frac{120}{3} = 40 \text{ mi/h}$$

So our guess of 45 mi/h was wrong.

SOLUTION

Understand the problem. ▶

We need to look more carefully at the meaning of average speed. It is defined as

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

Introduce notation. ▶

State what is given. ▶

Let d be the distance traveled on each half of the trip. Let t_1 and t_2 be the times taken for the first and second halves of the trip. Now we can write down the information we have been given. For the first half of the trip we have

$$30 = \frac{d}{t_1}$$

and for the second half we have

$$60 = \frac{d}{t_2}$$

Identify the unknown. ▶

Now we identify the quantity that we are asked to find:

$$\text{average speed for entire trip} = \frac{\text{total distance}}{\text{total time}} = \frac{2d}{t_1 + t_2}$$

Connect the given with the unknown. ▶

To calculate this quantity, we need to know t_1 and t_2, so we solve the above equations for these times:

$$t_1 = \frac{d}{30} \qquad t_2 = \frac{d}{60}$$

Now we have the ingredients needed to calculate the desired quantity:

$$\text{average speed} = \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{30} + \dfrac{d}{60}}$$

$$= \frac{60(2d)}{60\left(\dfrac{d}{30} + \dfrac{d}{60}\right)}$$

Multiply numerator and denominator by 60

$$= \frac{120d}{2d + d} = \frac{120d}{3d} = 40$$

So the average speed for the entire trip is 40 mi/h. ■

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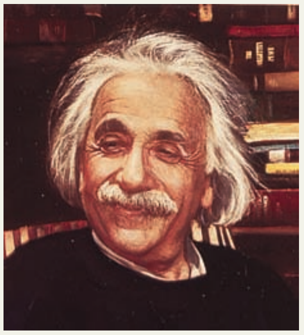


Bettmann/Corbis

Don't feel bad if you can't solve these problems right away. Problems 1 and 4 were sent to Albert Einstein by his friend Wertheimer. Einstein (and his friend Bucky) enjoyed the problems and wrote back to Wertheimer. Here is part of his reply:

> Your letter gave us a lot of amusement. The first intelligence test fooled both of us (Bucky and me). Only on working it out did I notice that no time is available for the downhill run! Mr. Bucky was also taken in by the second example, but I was not. Such drolleries show us how stupid we are!

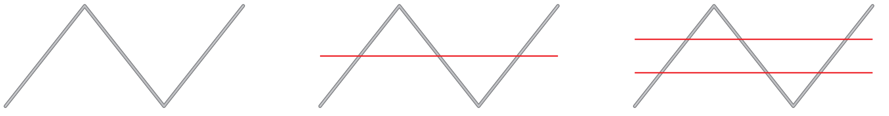
(See *Mathematical Intelligencer,* Spring 1990, page 41.)

PROBLEMS

1. **Distance, Time, and Speed** An old car has to travel a 2-mile route, uphill and down. Because it is so old, the car can climb the first mile—the ascent—no faster than an average speed of 15 mi/h. How fast does the car have to travel the second mile—on the descent it can go faster, of course—to achieve an average speed of 30 mi/h for the trip?

2. **Comparing Discounts** Which price is better for the buyer, a 40% discount or two successive discounts of 20%?

3. **Cutting up a Wire** A piece of wire is bent as shown in the figure. You can see that one cut through the wire produces four pieces and two parallel cuts produce seven pieces. How many pieces will be produced by 142 parallel cuts? Write a formula for the number of pieces produced by n parallel cuts.



4. **Amoeba Propagation** An amoeba propagates by simple division; each split takes 3 minutes to complete. When such an amoeba is put into a glass container with a nutrient fluid, the container is full of amoebas in one hour. How long would it take for the container to be filled if we start with not one amoeba, but two?

5. **Batting Averages** Player A has a higher batting average than player B for the first half of the baseball season. Player A also has a higher batting average than player B for the second half of the season. Is it necessarily true that player A has a higher batting average than player B for the entire season?

6. **Coffee and Cream** A spoonful of cream is taken from a pitcher of cream and put into a cup of coffee. The coffee is stirred. Then a spoonful of this mixture is put into the pitcher of cream. Is there now more cream in the coffee cup or more coffee in the pitcher of cream?

7. **Wrapping the World** A ribbon is tied tightly around the earth at the equator. How much more ribbon would you need if you raised the ribbon 1 ft above the equator everywhere? (You don't need to know the radius of the earth to solve this problem.)

8. **Ending Up Where You Started** A woman starts at a point P on the earth's surface and walks 1 mi south, then 1 mi east, then 1 mi north, and finds herself back at P, the starting point. Describe all points P for which this is possible. [*Hint:* There are infinitely many such points, all but one of which lie in Antarctica.]

Many more problems and examples that highlight different problem-solving principles are available at the book companion website: www.stewartmath.com. You can try them as you progress through the book.

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1 Fundamentals

In this first chapter we review the real numbers, equations, and the coordinate plane. You are probably already familiar with these concepts, but it is helpful to get a fresh look at how these ideas work together to solve problems and model (or describe) real-world situations.

In the *Focus on Modeling* at the end of the chapter we learn how to find linear trends in data and how to use these trends to make predictions about the future.

1.1 REAL NUMBERS

■ Real Numbers ■ Properties of Real Numbers ■ Addition and Subtraction ■ Multiplication and Division ■ The Real Line ■ Sets and Intervals ■ Absolute Value and Distance

In the real world we use numbers to measure and compare different quantities. For example, we measure temperature, length, height, weight, blood pressure, distance, speed, acceleration, energy, force, angles, age, cost, and so on. Figure 1 illustrates some situations in which numbers are used. Numbers also allow us to express relationships between different quantities—for example, relationships between the radius and volume of a ball, between miles driven and gas used, or between education level and starting salary.

FIGURE 1 Measuring with real numbers

■ Real Numbers

Let's review the types of numbers that make up the real number system. We start with the **natural numbers**:

$$1, 2, 3, 4, \ldots$$

The **integers** consist of the natural numbers together with their negatives and 0:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$$

We construct the **rational numbers** by taking ratios of integers. Thus any rational number r can be expressed as

$$r = \frac{m}{n}$$

where m and n are integers and $n \neq 0$. Examples are

$$\tfrac{1}{2} \qquad -\tfrac{3}{7} \qquad 46 = \tfrac{46}{1} \qquad 0.17 = \tfrac{17}{100}$$

(Recall that division by 0 is always ruled out, so expressions like $\frac{3}{0}$ and $\frac{0}{0}$ are undefined.) There are also real numbers, such as $\sqrt{2}$, that cannot be expressed as a ratio of integers and are therefore called **irrational numbers**. It can be shown, with varying degrees of difficulty, that these numbers are also irrational:

$$\sqrt{3} \qquad \sqrt{5} \qquad \sqrt[3]{2} \qquad \pi \qquad \frac{3}{\pi^2}$$

The different types of real numbers were invented to meet specific needs. For example, natural numbers are needed for counting, negative numbers for describing debt or below-zero temperatures, rational numbers for concepts like "half a gallon of milk," and irrational numbers for measuring certain distances, like the diagonal of a square.

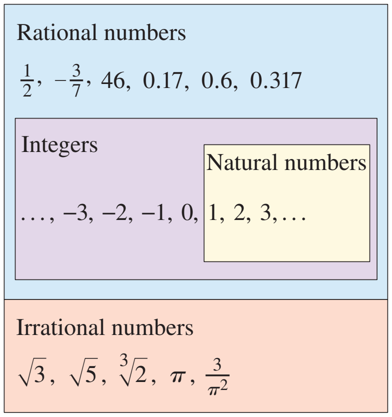

FIGURE 2 The real number system

The set of all real numbers is usually denoted by the symbol $\mathbb{R}$. When we use the word *number* without qualification, we will mean "real number." Figure 2 is a diagram of the types of real numbers that we work with in this book.

Every real number has a decimal representation. If the number is rational, then its corresponding decimal is repeating. For example,

$$\tfrac{1}{2} = 0.5000\ldots = 0.5\overline{0} \qquad \tfrac{2}{3} = 0.66666\ldots = 0.\overline{6}$$
$$\tfrac{157}{495} = 0.3171717\ldots = 0.3\overline{17} \qquad \tfrac{9}{7} = 1.285714285714\ldots = 1.\overline{285714}$$

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A repeating decimal such as

$$x = 3.5474747\ldots$$

is a rational number. To convert it to a ratio of two integers, we write

$$\begin{aligned} 1000x &= 3547.47474747\ldots \\ 10x &= 35.47474747\ldots \\ \hline 990x &= 3512.0 \end{aligned}$$

Thus $x = \frac{3512}{990}$. (The idea is to multiply x by appropriate powers of 10 and then subtract to eliminate the repeating part.)

(The bar indicates that the sequence of digits repeats forever.) If the number is irrational, the decimal representation is nonrepeating:

$$\sqrt{2} = 1.414213562373095\ldots \qquad \pi = 3.141592653589793\ldots$$

If we stop the decimal expansion of any number at a certain place, we get an approximation to the number. For instance, we can write

$$\pi \approx 3.14159265$$

where the symbol $\approx$ is read "is approximately equal to." The more decimal places we retain, the better our approximation.

Properties of Real Numbers

We all know that $2 + 3 = 3 + 2$, and $5 + 7 = 7 + 5$, and $513 + 87 = 87 + 513$, and so on. In algebra we express all these (infinitely many) facts by writing

$$a + b = b + a$$

where a and b stand for any two numbers. In other words, "$a + b = b + a$" is a concise way of saying that "when we add two numbers, the order of addition doesn't matter." This fact is called the *Commutative Property* of addition. From our experience with numbers we know that the properties in the following box are also valid.

PROPERTIES OF REAL NUMBERS

Property	Example	Description
Commutative Properties		
$a + b = b + a$	$7 + 3 = 3 + 7$	When we add two numbers, order doesn't matter.
$ab = ba$	$3 \cdot 5 = 5 \cdot 3$	When we multiply two numbers, order doesn't matter.
Associative Properties		
$(a + b) + c = a + (b + c)$	$(2 + 4) + 7 = 2 + (4 + 7)$	When we add three numbers, it doesn't matter which two we add first.
$(ab)c = a(bc)$	$(3 \cdot 7) \cdot 5 = 3 \cdot (7 \cdot 5)$	When we multiply three numbers, it doesn't matter which two we multiply first.
Distributive Property		
$a(b + c) = ab + ac$ $(b + c)a = ab + ac$	$2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5$ $(3 + 5) \cdot 2 = 2 \cdot 3 + 2 \cdot 5$	When we multiply a number by a sum of two numbers, we get the same result as we get if we multiply the number by each of the terms and then add the results.

The Distributive Property applies whenever we multiply a number by a sum. Figure 3 explains why this property works for the case in which all the numbers are positive integers, but the property is true for any real numbers a, b, and c.

The Distributive Property is crucial because it describes the way addition and multiplication interact with each other.

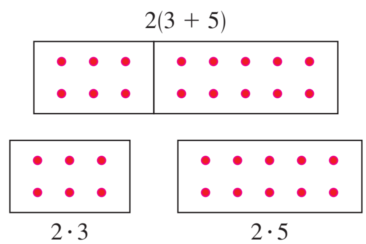

FIGURE 3 The Distributive Property

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